BioMedical Admissions Test (BMAT)

Section 2: Mathematics

Topic M3: Ratio and Proportion

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Ratio

Ratio notion

If there are x blue flowers and y pink flowers in a garden, then the ratio of blue flowers to pink flowers is $x : y$.

Simplifying ratios

To simplify ratios, you can divide or multiply both sides by the same number. This keeps the value of the ratio the same.

Simplifying basic ratios:

Example: Simplify 12:18
Divide both numbers by their high common factor: 6.
12:18 = 2:3
This is equivalent to the original ratio and therefore has the same value.

Simplifying ratios in the form of fractions:

Example: Give the ratio $\frac{8}{3} : \frac{7}{4}$ in its simplest form.
Convert both fractions into fractions with a common denominator - their lowest common multiple.
$\frac{8}{3} : \frac{7}{4} = \frac{32}{12} : \frac{21}{12}$

Multiply both sides by the common denominator.
$\frac{32}{12} : \frac{21}{12} = 32 : 21$

Divide both sides by their highest common factor to simplify if you can. In this case, this is the simplest form of the ratio.

Simplifying ratios in the form of decimals:

Example: Simplify the ratio 1.4 : 2.8
Multiply both sides by 10 to get rid of the decimal points.
1.4 : 2.8 = 14 : 28

Divide both sides by their highest common factor to simplify.
14 : 28 = 1: 2

Simplifying ratios with different units:

Make sure both sides of ratio share a common unit.

Example: Simplify ratio 0.5kg : 20g.
Convert to the smaller unit.
0.5kg = 500g

Divide both sides by their highest common factor to simplify.
500g : 20g = 25 : 1

Dividing a given ratio

Dividing a quantity, Q, in the ratio of $x:y$

1. **Calculate** $x+y$
2. **Divide** Q by $x+y$ (This finds the value of one unit)
3. **Multiply** the value from step 2 by x to find the value of x parts
4. **Multiply** the value from step 2 by y to find the value of y parts

This might seem confusing but the example below will demonstrate this.

Example: divide £250 in the ratio of 15:10

1. 15 + 10 = 25
2. 250 ÷ 25 = 10 (Each part is worth £10)
3. 15 x 10 = £150
4. 10 x 10 = £100

Tip: Make sure that you've done this right by quickly adding up the answers you got from steps 3 and 4 to make sure they add up to the original quantity given.

Ratios and fractions

If the ratio of $a:b$ is $x:y$, then $\frac{a}{b} = \frac{x}{y}$

Example:
In a jewellery box, there are n necklaces, r rings and e earrings. The ratio of $n:r$ is 2:3 and the ratio of $r:e$ is 4:5 What is the ratio of $n:e$?

Based on this information, we can form to equations using the rule that if $a:b$ is $x:y$, then $\frac{a}{b} = \frac{x}{y}$

$\frac{n}{r} = \frac{2}{3}$ (equation 1)

$\frac{r}{e} = \frac{4}{5}$ (equation 2)

So to find $n:e$, we must find $\frac{n}{e}$

First we multiply the numerator and denominator by r, as they are found in both equations 1 and 2. Then we rearrange the fractions so that we can substitute in our two equations.

$$\frac{n}{e} = \frac{n\times r}{e\times r}$$
$$= \frac{n}{r} \times \frac{r}{e}$$
$$= \frac{2}{3} \times \frac{4}{5}$$
$$= \frac{8}{15}$$

So if $\frac{n}{e} = \frac{8}{15}$, then $n : e$ = 8:15.

Comparing lengths and volumes using ratios

If two shapes are **mathematically similar** it means that one shape is an enlargement of the other. Therefore, the angles remain the same and the lengths of the sides remain in the same ratio.

Linear scale factor:

If shape A and shape B are mathematically similar and we are told the length of B's sides are equal to x times the length of A's corresponding sides, it follows that:

The area of shape B is x^2 times the area of shape A

The volume of shape B is x^3 times the volume of shape A

Linear ratio:

If shape A and shape B are mathematically similar, and we are told that the length of sides of A to B are in the ratio $x : y$, then we can say that:

The ratio of the corresponding areas is $x^2 : y^2$

The ratio of the volumes is $x^3 : y^3$

Area ratio:

If shape A and shape B are mathematically similar, and we are told that the area of B is $x : y$ times the area of A, then we can say that:

The lengths of B are $\sqrt{x} : \sqrt{y}$ times the equivalent lengths of shape A

Volume ratio:

If shape A and shape B are mathematically similar, and we are told that the volume of B is x times the volume of A, then we can say that:

The lengths of B are $\sqrt[3]{x}$ times the equivalent lengths of shape A

The area of shape B is $(\sqrt[3]{x})^2$ times the area of shape A

Trigonometric ratio

Ratios can also be used in trigonometry. As we mentioned before, if two shapes are mathematically similar, then the angles stay the same - it is only the relative lengths of the sides that change. Therefore, even if a triangle is scaled up or down, **the angles at each point remain unchanged**.

Proportion

If x number of books cost y pounds, then 1 book would cost $\frac{y}{x}$ pounds.

If the ratio between some toys in a box is $a : b$, for a toy A and toy B respectively, then the fraction of toys A to the total toys in the box is $\frac{a}{a+b}$. This is as the total number of toys is $a + b$

Percentages

Percentage means 'number of parts per hundred'. So if 3 out of 10 sweets are red, then this is equivalent to 30 out of 100. This can be written as 30% of the sweets being red.

Calculating a percent of a value

The simplest way to find the percent of a value, is to first convert the percentage to either a fraction or decimal (the choice is yours).

Example: 40% of £300.

Using fractions, we can convert this to $\frac{40}{100} \times 300 = \frac{4}{10} \times 300 = \frac{1200}{10} = 120$

Using decimals, we can convert this to 0.4 x 300 = 40 x 3 = 120

Expressing value as a percentage

First write the question as a fraction. Then equate the fraction into a percentage.

Example: express 15 as a percentage of 250.

$\frac{15}{250} = \frac{3}{50}$

$\frac{3}{50} \times 100 = \frac{300}{50} = 6\%$

Percentage increase/decrease

Percentage change = $\frac{change}{original} \times 100$

Example: A T-shirt used to cost £12, but in the sale the T-shirt is sold for £8. What percentage was saved by the customer?

Percentage change = $\frac{12-8}{12} \times 100 = \frac{4}{12} \times 100 = \frac{1}{3} \times 100 = 33.3\%$

Finding the original value

Treat the original value that we are trying to calculate as 100%. If the price was decreased by x%, then the new price is (100-x)%. If we know that

Original price = $\frac{Q}{100-x} \times 100$ where Q is the new price and x is the % reduction

Original price = $\frac{Q}{100+y} \times 100$ where Q is the new price and y is the % increase

Example: This month the price of fruits at the local market went up by 50%. I bought the fruits for £3.75. How much did I used to pay before the price increase?

Treat last month's (the original) price as 100%.

If it has been a 50% increase, then the new price is 150%.

Original price = $\frac{Q}{100+y} \times 100$

= $\frac{3.75}{150} \times 100$ = $= \frac{375}{150} = \frac{375}{150} = £2.50$

Simple interest

Interest = $\frac{PRT}{100}$ where P=principal sum, R=interest rate, T=time

Example: You put £300 in your savings account and the simple interest is 2% per annum. After 10 years, how much interest will you have received?

Interest = $\frac{PRT}{100} = \frac{300 \times 2 \times 10}{100} = \frac{6000}{100} = £60$

Compound interest

Final value = $P(1 + \frac{R}{100})^t$ where P=principal sum, R=interest rate, T=time

Example: You put £1000 into a savings account with a compound interest of 5% per annum. You leave the money for 3 years. How much interest have you received?

Final value = $1000(1 + \frac{5}{100})^3 = 1000 \times 1.05^3 = £1157.63$ (to the nearest penny)
Therefore, interest earned is £1157.63 - £1000 = £157.63

Compound Growth and Decay

The base value (principal) used to calculate interest/growth/decay changes every time period to reflect additional value (interest/growth/decay) gained within that particular period

Final Value = $x_0(r)^t$ where x_0 = initial value, *r* = change per unit time, *t* = time

Example: In a town, the number of people infected by a new disease doubles each day. At the end of day 1 there are 100 people infected. How many people have caught the disease by the end of the fifth day?

The end of the fifth day means that there have been only 4 days since 100 people were infected.

Final value = $100(2)^4 = 100 \times 2^4 = 1600$ people infected on day 5.

Example 2: At the end of day 5, a cure if found. The number of infected patients decreases exponentially. After 3 days of distributing the cure, there are only 200 patients left. How many patients will be left at the end of the fifth day?

Final Value = $x_0(r)^t$

200 = $1600(r)^2$

$r^2 = \frac{200}{1600} = \frac{1}{8}$

$r = \sqrt{\frac{1}{8}}$

After 5 days of the cure, there will be 1600r^4 = 1600($\sqrt{\frac{1}{8}}$)^4 = 1600$(\frac{1}{64}) = 25$